Classical and Quantum Field Theories from Hamiltonian Constraint

Václav Zatloukal
[www.zatlovac.eu]

Faculty of Nuclear Sciences and Physical Engineering,
Czech Technical University in Prague

and

Freie Universität and MPI for the History of Science, Berlin

Rethinking Foundations of Physics
Dorfgastein, Austria
March 2016

Motivation

Consider a non-relativistic mechanical system with Hamiltonian $H_0(\mathbf{x}, \mathbf{p})$:

Canonical equations of motion:

$$\frac{d\mathbf{x}}{dt} = \frac{\partial H_0}{\partial \mathbf{p}} \quad , \quad \frac{d\mathbf{p}}{dt} = -\frac{\partial H_0}{\partial \mathbf{x}} \tag{1}$$

Hamilton-Jacobi equation: $S(\mathbf{x}, t)$

$$\frac{\partial S}{\partial t} + H_0(\mathbf{x}, \frac{\partial S}{\partial \mathbf{x}}) = 0 \tag{2}$$

Quantization & Schrödinger equation: $\mathbf{p} \to -i\hbar\,\partial/\partial\mathbf{x}$

$$\left[-i\hbar\frac{\partial}{\partial t} + H_0(\mathbf{x}, -i\hbar\frac{\partial}{\partial \mathbf{x}})\right]\psi(\mathbf{x}, t) = 0 \tag{3}$$

Motivation

Our goal: **Hamiltonian formulation of field theory**

Today's presentation: Classical field theory
(generalized: momentum, canonical equations, Hamilton-Jacobi theory)
[V. Zatloukal, arXiv:1504.08344 (2015), arXiv:1602.00468 (2016)]

Discussion: Quantization
(generalized: momentum operator, wavefunctions, Schrödinger equation)

Outline

- Geometric algebra formalism
- Partial observables and Relativistic configuration space
- Variational principle with Hamiltonian constraint
- Canonical equations of motion
- Local Hamilton-Jacobi theory
- Symmetries and Hamiltonian Noether theorem
- Examples:
  - Non-relativistic Hamiltonian mechanics
  - Scalar field theory
  - String theory
- Discussion: Quantization

Geometric algebra formalism

We use the mathematical formalism of **geometric algebra and calculus**:
[D. Hestenes and G. Sobczyk, *Clifford Algebra to Geometric Calculus*, (1987)]
See also [C. Doran and A. Lasenby, *Geometric Algebra for Physicists*, (2007)]
($\Leftrightarrow$ Clifford algebra, Dirac algebra of γ-matrices)

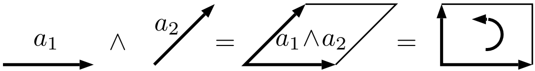
Geometric product: a, b ... vectors in an n-dim. vector space

$$ab = a \cdot b + a \wedge b \tag{4}$$

– associative, invertible, non-commutative

($\cdot$) **inner product** (grade-lowering)
– non-associative, $a \cdot b = b \cdot a$

($\wedge$) **outer product** (grade-raising)
– associative, $a \wedge b = -b \wedge a$



Vectors $a_1, \ldots, a_D$ $\rightarrow$ multivector $a_1 \wedge \ldots \wedge a_D$ of grade D.

Geometric algebra formalism

Generic multivector A: a sum of terms with various grades

Geometric algebra $\mathcal{G}$ ... space of A's endowed with the geometric product

Orthonormal basis $\{e_j\}$ ($e_j \cdot e_k = \delta_{jk}$)

$$\rightarrow \quad \mathcal{G} = \mathrm{span}\{\underbrace{1}_{scalar}, \underbrace{e_j}_{vectors}, \underbrace{e_j e_k}_{bivectors}, \ldots, \underbrace{e_1 \ldots e_n}_{pseudoscalar}\} \quad , \quad e_J \equiv e_{j_1} \ldots e_{j_D} \tag{5}$$

Reversion: $\widetilde{AB} = \widetilde{B}\widetilde{A}$, $\widetilde{a} = a \quad \rightarrow \quad \widetilde{A}_D = (-1)^{D(D-1)/2} A_D$

Magnitude: $|A| := \sqrt{\langle \widetilde{A} A \rangle}$, $\langle \ldots \rangle$ ... scalar part

Priority: $a \cdot A\,B = (a \cdot A)B$, $a \wedge A\,B = (a \wedge A)B$

Differential forms: D-vector $A \rightarrow$ scalar function α

$$\alpha(b_1, \ldots, b_D) := \widetilde{A} \cdot (b_1 \wedge \ldots \wedge b_D) \tag{6}$$

Geometric calculus formalism

$q \in \mathrm{manifold}\ \mathcal{C}$ (Euclidean space)

Vector derivative: $a \cdot \partial_q$ ... derivative in direction a

$$\partial_q F(q) := \sum_{j=1}^{N+D} e_j (e_j \cdot \partial_q) F(q) = \underbrace{\partial_q \cdot F}_{divergence} + \underbrace{\partial_q \wedge F}_{curl} \quad (7)$$

Leibniz rule: $\partial_q(FG) = \dot{\partial}_q \dot{F} G + \dot{\partial}_q F \dot{G}$

Transformation $q' = f(q)$:

differential outermorphism:

$$\underline{f}(a; q) \equiv a \cdot \partial_q f(q) \quad , \quad \underline{f}(A \wedge B) = \underline{f}(A) \wedge \underline{f}(B) \quad (8)$$

adjoint:

$$\overline{f}(b; q) \equiv \partial_q f(q) \cdot b \quad \rightarrow \quad b \cdot \underline{f}(a) = \overline{f}(b) \cdot a \quad (9)$$

Geometric calculus formalism

Integration: $\gamma \subset \mathcal{C}$

$$\int_\gamma F(q)\, d\Gamma(q)\, G(q) := \lim_{n\to\infty} \sum_{i=1}^{n} F(q_i)\, \Delta\Gamma(q_i)\, G(q_i) \qquad (10)$$

Fundamental theorem of geometric calculus: (generalized Stokes theorem)

$$\int_\gamma \dot{F}\, d\Gamma \cdot \dot{\partial}_q\, \dot{G} = \int_{\partial\gamma} F\, d\Sigma\, G \qquad (11)$$

Multivector derivative: $A, P\ \ldots\ D$-vectors

$$A \cdot \partial_P F(P) := \lim_{\varepsilon\to 0} \frac{F(P+\varepsilon A) - F(P)}{\varepsilon} \qquad (12)$$

$$\partial_P F(P) := \sum_{|J|=D} \widetilde{e}_J (e_J \cdot \partial_P) F(P) \qquad (13)$$

Partial observables and Relativistic configuration space

Non-relativistic mechanics:

Hamiltonian $H_0(\mathbf{x}, \mathbf{p})$

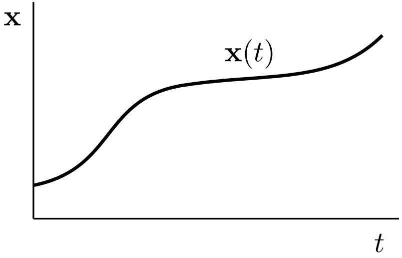
Trajectories are functions $\mathbf{x}(t)$

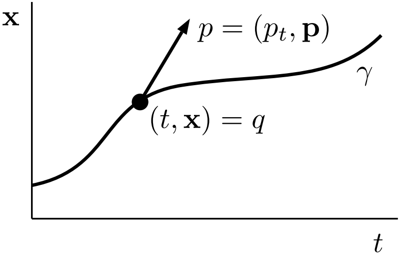
Relativistic formalism:

Curves $\gamma = \{q = (t, \mathbf{x}) \,|\, f(t, \mathbf{x}) = 0\}$

Hamiltonian constraint

$H(q, p) = p_t + H_0(\mathbf{x}, \mathbf{p}) = 0$

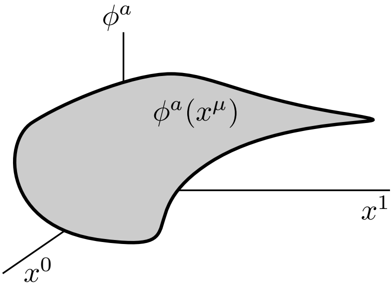


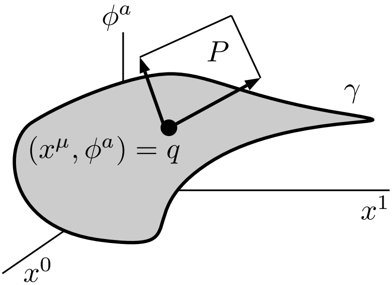


Relativistic formalism is more compact, symmetric, and allows to describe both non-relativistic and relativistic mechanical systems (e.g., free relativistic particle: $H = p_\mu p^\mu - m^2$).

Partial observables and Relativistic configuration space

Field theory: functions $\phi^a(x^\mu) \to$ surfaces $\gamma = \{q = (x^\mu, \phi^a) \,|\, f(x, \phi) = 0\}$

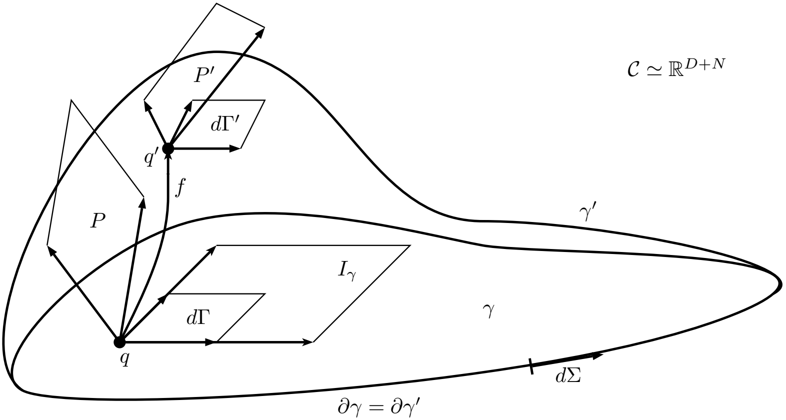




Following [C. Rovelli, *Quantum Gravity*, Cambridge Univ. Press (2004), Ch. 3]

$t, \mathbf{x}, \phi$ … *partial observables*
$\mathcal{C} = \{q\}$ … *configuration space* – $N + D$-dimensional, Euclidean
$\gamma \subset \mathcal{C}$ … *motions* – D-dim., correlations among partial observables

Variational principle with Hamiltonian constraint



$d\Gamma$ ... oriented surface element of γ
P ... multivector of grade D

Variational principle with Hamiltonian constraint

Variational principle

A surface $\gamma_{\rm cl}$ with boundary $\partial\gamma_{\rm cl}$ is a physical motion, if the couple $(\gamma_{\rm cl}, P_{\rm cl})$ extremizes the (action) functional

$$\mathcal{A}[\gamma, P] = \int_\gamma P(q) \cdot d\Gamma(q) \qquad (14)$$

in the class of pairs (γ, P), for which $\partial\gamma = \partial\gamma_{\rm cl}$, and P defined along γ satisfies the **Hamiltonian constraint**

$$H(q, P(q)) = 0 \qquad \forall q \in \gamma. \qquad (15)$$

(cf. Ch. 3.3.2 in [C. Rovelli, Quantum Gravity, Cambridge Univ. Press (2004)])

Non-relativistic mechanics ... $H = p \cdot e_t + H_0(q, p_x)$
Scalar field theory ... $H = P \cdot I_x + \frac{1}{2}\sum_{a=1}^N \left(I_x \cdot (P \cdot e_a)\right)^2 + V(y)$
String theory ... $H = \frac{1}{2}(|P|^2 - \Lambda^2)$

Canonical equations of motion

Extended action:

$$\mathcal{A}[\gamma, P, \lambda] = \int_\gamma [P(q) \cdot d\Gamma(q) - \lambda(q) H(q, P(q))] \qquad (16)$$

Lagrange multiplier $\lambda(q)$ – infinitesimal ($\lambda \sim |d\Gamma|$)

Variation with respect to γ, P, λ yields:

(see [V. Zatloukal, arXiv:1504.08344 (2015)] for detailed derivation)

Canonical equations of motion

Canonical equations of motion

Physical motions γ_{cl} are obtained by solving the system of equations

$$\lambda\,\partial_P H(q,P) = d\Gamma, \tag{17a}$$

$$(-1)^D \lambda\,\dot{\partial}_q H(\dot{q},P) = \begin{cases} d\Gamma\cdot\partial_q P & \text{for } D=1 \\ (d\Gamma\cdot\partial_q)\cdot P & \text{for } D>1, \end{cases} \tag{17b}$$

$$H(q,P) = 0. \tag{17c}$$

(17a) "Velocity–momentum" relation
(17b) "Force = Change in momentum"
(17c) Hamiltonian constraint

Local Hamilton-Jacobi theory

Suppose $P(q) = \partial_q \wedge S(q)$ on an open subset of $\mathcal{C}$, for a $D-1$-vector S

IF (see Eq. (17c))

Local Hamilton-Jacobi equation

$$H(q, \partial_q \wedge S) = 0, \tag{18}$$

AND (see Eq. (17a))

$$\lambda\, \partial_P H(q, \partial_q \wedge S) = d\Gamma, \tag{19}$$

THEN

the second canonical equation (17b) is fulfilled automatically.

Local Hamilton-Jacobi theory

If we find a family of solutions $S(q;\alpha)$, where α is a continuous parameter, by differentiation ∂_α we obtain:

$D = 1$: Constant of motion

$$d\Gamma \cdot \partial_q(\partial_\alpha S) = 0 \quad \Rightarrow \quad \partial_\alpha S(q;\alpha) = \beta \qquad \forall q \in \gamma_{\rm cl}, \tag{20}$$

With N independent parameters $\alpha_1, \ldots, \alpha_N$, we determine $\gamma_{\rm cl}$ from implicit equations (20). (Note: $\mathcal{C} \simeq \mathbb{R}^{N+1}$)

$D > 1$: Continuity equation

$$(d\Gamma \cdot \partial_q) \cdot (\partial_\alpha S) = 0 \quad \Rightarrow \quad \int_{\bar\gamma_{\rm cl}} (d\Gamma \cdot \partial_q) \cdot (\partial_\alpha S) = \int_{\partial\bar\gamma_{\rm cl}} d\Sigma \cdot (\partial_\alpha S) = 0 \tag{21}$$

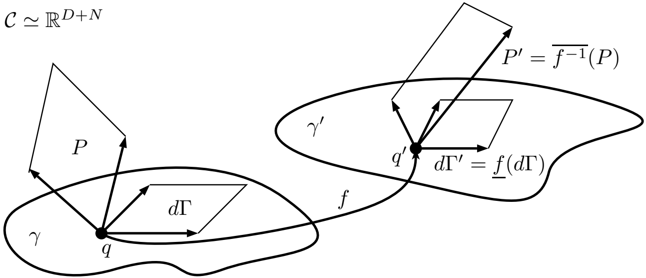
where $\bar\gamma_{\rm cl}$ is in general a subset of $\gamma_{\rm cl}$.

Symmetries and Hamiltonian Noether theorem

Transformation $q' = f(q)$:

$$\gamma' = \{f(q) \,|\, q \in \gamma\} \quad , \quad d\Gamma'(q') = \underline{f}(d\Gamma(q); q) \quad , \quad P' = \overline{f^{-1}}(P; q) \quad (22)$$

$$\Rightarrow \quad \mathcal{A}[\gamma', P'] = \mathcal{A}[\gamma, P] \quad (23)$$



f is a symmetry if: $H(q', P') = H(q, P)$
(Then classical motions are mapped to classical motions.)

Symmetries and Hamiltonian Noether theorem

Infinitesimal symmetry $f(q) = q + \varepsilon v(q)$:

$$v \cdot \dot{\partial}_q H(\dot{q}, P) - \left(\dot{\partial}_q \wedge (\dot{v} \cdot P)\right) \cdot \partial_P H(q, P) = 0 \tag{24}$$

$\oplus$ Canonical equations $\Rightarrow$

Conservation law

$$0 = \begin{cases} d\Gamma \cdot \partial_q (P \cdot v) & \text{for } D = 1 \\ (d\Gamma \cdot \partial_q) \cdot (P \cdot v) & \text{for } D > 1 \end{cases} \tag{25}$$

Integral form:

$$P(q_2) \cdot v(q_2) = P(q_1) \cdot v(q_1) \qquad \text{resp.} \qquad \int_{\partial \gamma_{\rm cl}} d\Sigma \cdot (P \cdot v) = 0 \tag{26}$$

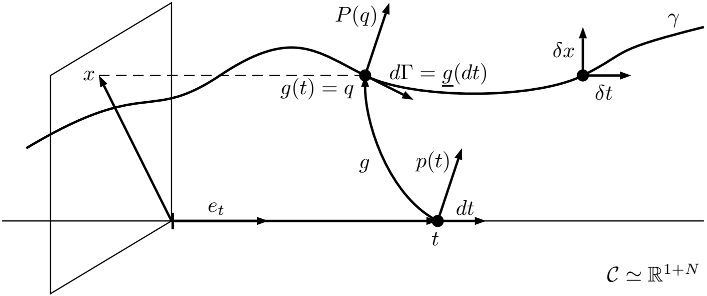
$P \cdot v$ ... conserved multivector of grade $D - 1$ ($\sim$ Noether current)

Example 1: Non-relativistic Hamiltonian mechanics

Consider $D = 1$, split $\mathcal{C} = \mathrm{time} \oplus \mathrm{space}$ ($q = t + x$), and take

$$H_{NR}(q, p) = p \cdot e_t + H_0(q, p_x), \qquad (27)$$

H_0 ... non-relativistic Hamiltonian,
p_x ... spatial part of p.



$$\gamma = \{q = t + x(t) \,|\, t \in \mathrm{span}\{e_t\}\} \quad , \quad \mathrm{p}(t) \equiv p(t + x(t)) \qquad (28)$$

Example 1: Non-relativistic Hamiltonian mechanics

Canonical eqs. (17) $\Rightarrow$ Hamilton's canonical equations:

$$e_t \cdot \partial_t x = \partial_{p_x} H_0 \quad , \quad e_t \cdot \partial_t \mathrm{p}_x = -\partial_x H_0 \tag{29}$$

Hamilton-Jacobi equation: ($S(q)$ is scalar function)

$$H_{NR}(q, \partial_q S) = e_t \cdot \partial_t S + H_0(q, \partial_x S) = 0 \tag{30}$$

Constants of motion:

1) $p \cdot e_t = -H_0$ ... symmetry generator $v = e_t$ [condition $e_t \cdot \partial_q H_0 = 0$]

2) $p_x \cdot v_x$ ... $v = v_x(x)$ [$v_x \cdot \partial_x H_0 - (\dot{\partial}_x \dot{v}_x \cdot p_x) \cdot \partial_p H_0 \equiv \underbrace{\{H_0, p_x \cdot v_x\}}_{\textit{Poisson bracket}} = 0$]

Example 2: Scalar field theory

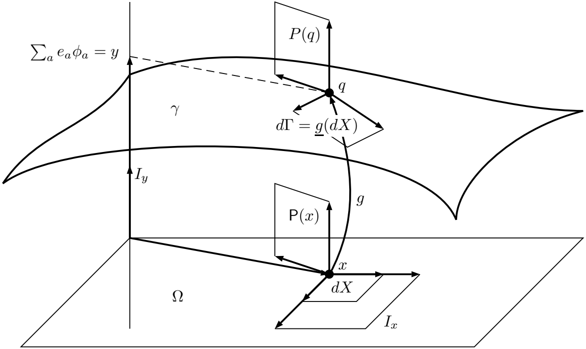
Consider $D > 1$, split $\mathcal{C} = \text{spacetime} \oplus \text{field space}$ ($q = x + y$), and take

$$H(q, P) = P \cdot I_x + H_{DW}(q, P). \quad (31)$$

H_{DW} ... De Donder-Weyl Hamiltonian, satisfying

$$I_x \cdot \partial_P H_{DW} = 0 \quad , \quad (e_b \wedge e_a) \cdot \partial_P H_{DW} = 0. \quad (32)$$

($\{e_a\}_{a=1}^N$ ... orthonormal basis of the field space)



Example 2: Scalar field theory

$$\gamma = \{q = x + y(x) \,|\, x \in \Omega\} \quad , \quad \mathrm{P}(x) \equiv P(x + y(x)) \tag{33}$$

Canonical eqs. (17) $\Rightarrow$ De Donder-Weyl equations:

$$\partial_x y = I_x^{-1} \partial_P H_{DW} \quad , \quad (e_a I_x \partial_x) \cdot \mathrm{P} = (-1)^D e_a \cdot \partial_y H_{DW} \tag{34}$$

(cf. [I. V. Kanatchikov, Rep. Math. Phys. **41**, 49 (1998)])

Hamilton-Jacobi equation:

$$I_x \cdot (\partial_q \wedge S) + H_{DW}(q, \partial_q \wedge S) = 0 \tag{35}$$

For $S(q) = s(q) \cdot I_x^{-1} \Rightarrow$ Weyl's eq. [H. Kastrup, Phys. Rep. **101**, 1-167 (1983)].

Example 2: Scalar field theory

Scalar field Hamiltonian:

$$H_{SF}(q,P) = P \cdot I_x + \frac{1}{2}\sum_{a=1}^{N}\big(I_x \cdot (P \cdot e_a)\big)^2 + V(y) \tag{36}$$

First canonical eq. (17a) $\Rightarrow$ Action, Eq. (16), reads

$$\mathcal{A}_{SF} = \int_\Omega \{\mathrm{P} \cdot [dX + (dX \cdot \partial_x) \wedge y] - |dX| H_{SF}\} = \int_\Omega \mathcal{L}_{SF}(\phi_a, \partial_x \phi_a)\,|dX| \tag{37}$$

where $\phi_a \equiv e_a \cdot y$, and the Lagrangian

$$\mathcal{L}_{SF}(\phi_a, \partial_x \phi_a) = \frac{1}{2}\sum_{a=1}^{N}(\partial_x \phi_a)^2 - V(y) \tag{38}$$

(N-component real scalar field theory)

Example 2: Scalar field theory

$\mathrm{v}(x) \equiv v(x + y(x))$

Conservation law

$$(d\Gamma \cdot \partial_q) \cdot (P \cdot v) = 0 \tag{39}$$

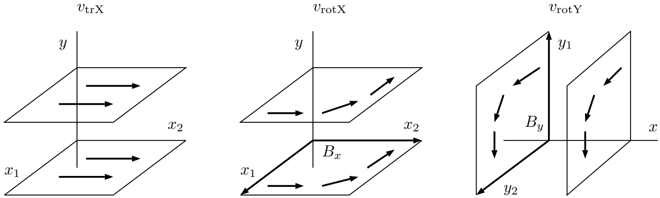
$\Rightarrow$ Continuity equation

$$\partial_x \cdot j(x) = 0 \tag{40}$$

Noether current

$$j(x) \equiv -I_x \cdot \left[\mathrm{P} \cdot \mathrm{v} + \dot{\partial}_x \wedge \left(\dot{y} \cdot (\mathrm{P} \cdot \mathrm{v})\right)\right] \tag{41}$$

Example 2: Scalar field theory symmetries



1) Translations in spacetime: $v_{\mathrm{trX}}(q) = v_x$ → energy-momentum tensor

$$j_{\mathrm{trX}}(x; v_x) = -v_x \mathcal{L}_{SF} + \sum_{a=1}^{N} (v_x \cdot \partial_x \phi_a) \frac{\partial \mathcal{L}_{SF}}{\partial(\partial_x \phi_a)} \tag{42}$$

2) Rotations in spacetime: $v_{\mathrm{rotX}}(q) = (q - x_0) \cdot B_x$ → angular momentum

$$j_{\mathrm{rotX}}(x; B_x, x_0) = j_{\mathrm{tr}}\big(x; (x - x_0) \cdot B_x\big) \tag{43}$$

3) Rotations in field space: $v_{\mathrm{rotY}}(q) = q \cdot B_y$

$$j_{\mathrm{rotY}}(x; B_y) = \sum_{a,b=1}^{N} (e_a \wedge e_b) \cdot B_y \, \phi_a \partial_x \phi_b \tag{44}$$

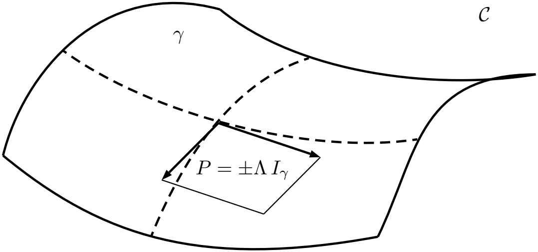
Example 3: String theory

$\mathcal{C}$ ... target space (Euclidean), dim. $N + D$
γ ... world-sheet, dim. D

Hamiltonian:

$$H_{Str}(P) = \frac{1}{2}(|P|^2 - \Lambda^2) \quad (45)$$

where $|P|^2 \equiv \widetilde{P} \cdot P$.



Canonical Eqs. (17) imply:

$$d\Gamma = \lambda \widetilde{P} \quad , \quad |d\Gamma| = |\lambda|\Lambda$$

$I_\gamma \equiv d\Gamma/|d\Gamma| = \pm P/\Lambda$ ... unit pseudoscalar of γ

Example 3: String theory

$D = 1$: Relativistic particle

$$I_\gamma \cdot \partial_q I_\gamma = 0 \tag{46}$$

$D > 1$: String or membrane

$$(I_\gamma \cdot \partial_q) \cdot I_\gamma = 0 \tag{47}$$

Hamilton-Jacobi equation:

$$|\partial_q \wedge S| = \Lambda \tag{48}$$

Symmetries:

$$v(q) = v_0 + q \cdot B_0 \tag{49}$$

(translations in direction $v_0 \oplus$ rotations in plane B_0)

Conserved quantities: $P \cdot v = \pm\Lambda \widetilde{I}_\gamma \cdot v$

Example 3: String theory

Nambu-Goto action:

$$\mathcal{A}_{Str} = \int_\gamma P \cdot d\Gamma = \int_\gamma \frac{1}{\lambda}|d\Gamma|^2 = \pm\Lambda \int_\gamma |d\Gamma| \tag{50}$$

$\rightarrow \gamma_{\mathrm{cl}}$ is a *minimal surface* (mean curvature vanishes)

Scalar field limit: worldsheet flattening

$$\gamma = \{q = x + y(x) \,|\, x \in \Omega\} \quad , \quad d\Gamma \approx dX + (dX \cdot \partial_x) \wedge y \tag{51}$$

$$\mathcal{A}_{Str} \approx \pm\Lambda\,\mathcal{A}_{SF}|_{V=0} \pm \Lambda \int_\Omega |dX| \tag{52}$$

String theory $\rightarrow$ Potential-free massless scalar field theory.
(cf. Relativistic free particle $\rightarrow$ Non-relativistic free particle)

Example 3: Relativistic particle – classical motions

$(D = 1)$

Integrating ($|d\Gamma|$-multiple of) Eq. (46) along γ from q_0 to q, and applying the *Fundamental theorem of geometric calculus*,

$$0 = \int_{q_0}^{q} d\Gamma \cdot \partial_q I_\gamma = I_\gamma(q) - I_\gamma(q_0) \qquad (53)$$

$\Rightarrow$ I_γ is constant along a classical motion
$\Rightarrow$ $\gamma_{\rm cl}$ are straight lines in $\mathcal{C}$:

$$\gamma_{\rm cl} = \{q = w\tau + q_0 \,|\, \tau \in \mathbb{R}\} \qquad (54)$$

($q_0 \in \mathcal{C}$ and w is an arbitrary constant vector.)

Summary of results

- We have seen how field theory can be formulated using Hamiltonian constraint between partial observables and generalized momentum:
  $\mathcal{A} = \int_\gamma P \cdot d\Gamma \quad , \quad H(q, P) = 0$

- Canonical equations of motion:
  $\lambda\, \partial_P H(q, P) = d\Gamma \quad , \quad (-1)^D \lambda\, \dot{\partial}_q H(\dot{q}, P) = (d\Gamma \cdot \partial_q) \cdot P$

- Local Hamilton-Jacobi equation:
  $H(q, \partial_q \wedge S) = 0$

- Field-theoretic Hamiltonian Noether theorem:
  $(d\Gamma \cdot \partial_q) \cdot (P \cdot v) = 0$

- Three examples provided:
  Non-relativistic mechanics, Scalar field theory, String theory

Discussion: Symmetry between spacetime and field space

Hamiltonian constraint formulation of **mechanics** – double significance:

1) formal: More general than non-relativistic Hamiltonian mechanics. Equations take compact and symmetric form (e.g., Hamilton-Jacobi eq.).

2) physical: Allows to formulate special relativity – a physical theory of utmost importance.

Hamiltonian constraint formulation of **field theory**:

$\rightarrow$ 1) General framework for various theories (e.g., scalar field, string theory). Provides insights, and neatly derives relevant equations.

$\rightarrow$ 2) Should the field and the spacetime coordinates be put on the same footing? (In gravity, the spacetime is dynamical – a kind of field?)

Discussion: Quantization – path integral

Mechanics ($D = 1$):

$$\psi(q) \equiv \langle q|q_0\rangle = \int_{q_0}^{q} \mathcal{D}q\mathcal{D}p \, e^{\frac{i}{\hbar}\int_{q_0}^{q} p\cdot dq} \delta[H(q,p)] \qquad (55)$$

$\rightarrow$ differential equation:

$$\psi(q) = \delta\big(H(q, -i\hbar\partial_q)\big)\psi(q) \quad \overset{\alpha\delta(\alpha)=0}{\Rightarrow} \quad H(q, -i\hbar\partial_q)\psi(q) = 0 \qquad (56)$$

Schrödinger eq. for $H = H_{NR}$ (Eq. 27)
Klein-Gordon eq. for $H = H_{Str}$ (Eq. 45)

Field theory ($D > 1$): $\psi[\partial\gamma]$ ... functional of the boundary

$$\psi[\partial\gamma] = \int_{\partial\gamma \text{ fixed}} \mathcal{D}\gamma\mathcal{D}P \, e^{\frac{i}{\hbar}\int_{\gamma} P\cdot d\Gamma} \delta[H(q,P)] \qquad (57)$$

$\rightarrow$ functional differential equation: (?)

Discussion: Quantization – "canonical"

Mechanics: Hamilton-Jacobi eq. $\rightarrow$ Schrödinger eq.

$$H(q, \partial_q S(q)) = 0 \quad \rightarrow \quad H(q, -i\hbar\partial_q)\psi(q) = 0 \tag{58}$$

classical momentum $\rightarrow$ quantum operator

$$p \quad \rightarrow \quad \hat{p} = -i\hbar\partial_q \tag{59}$$

Field theory:

Local Hamilton-Jacobi eq. (18) $\rightarrow$ partial differential equation

$$H(q, \partial_q \wedge S(q)) = 0 \quad \rightarrow \quad (?) \tag{60}$$

classical momentum D-vector $\rightarrow$ quantum operator

$$P \quad \rightarrow \quad \hat{P} = \ (?) \tag{61}$$

(Hints in [I. V. Kanatchikov, arXiv:1312.4518 (2013)])

Thank you for your attention.